Answers to Maths Advance - Level 9+ Exam Paper

Q1.

$$A + B = \left(2^2 \times 3 \times 13 \times 17^2\right) + \left(2^3 \times 5 \times 13 \times 17\right)$$

$$= 2^2 \times 13 \times 17 \times ((3 \times 17) + (2 \times 5))$$

$$= 2^2 \times 13 \times 17 \times (61)$$

$$= 2^2 \times 13 \times 17 \times 61$$

Q2.

$$\frac{3^{111} - 3^{110}}{3^{109} + 3^{107}} = \frac{3^{110}(3-1)}{3^{107}(3^2-1)}$$

$$= \frac{3^3 \times 2}{8}$$

$$= \frac{27}{4}$$

Q3.

$$\left(\sqrt{a}+\sqrt{b}\right)^2 = a + 2\sqrt{ab} + b$$

$$= (a+b) + \sqrt{4ab}$$

Comparing this expression to $5+\sqrt{24}$ gives:

$a+b=5$

$ab=6$

This solves to give $(a,b)=(2,3)$ or $(3,2)$

Thus $\sqrt{5+\sqrt{24}}=\sqrt{2}+\sqrt{3}$

Q4.

George must have rolled three 1's, one 2, and one 5.

There are therefore twenty different ways in which he could have rolled the dice.

Q5.

First bar = 104

Second bar = 148

Third bar = 144

Fourth bar = 132

Fifth bar = 48

Scale on the y-axis is...

The large squares equate to frequency densities of 2, 4, 6 etc.

The smaller squares equate to frequency densities of 0.4, 0.8, 1.2 etc.

Q6.

Height of triangle:

$$h^2 = a^2 - \left(\tfrac{1}{2}a\right)^2$$

$$= \tfrac{3}{4}a^2$$

$$h = \tfrac{\sqrt{3}}{2}a$$

Apothem (A) of hexagon:

$$A = a - \tfrac{\sqrt{3}}{2}a$$

$$= \tfrac{2-\sqrt{3}}{2}a$$

Side length (S) of hexagon:

$$s^2 - \left(\tfrac{1}{2}s\right)^2 = \left(\tfrac{2-\sqrt{3}}{2}a\right)^2$$

$$\tfrac{3}{4}s^2 = \frac{\left(2-\sqrt{3}\right)^2 a^2}{4}$$

$$s^2 = \frac{\left(2-\sqrt{3}\right)^2 a^2}{3}$$

Area of hexagon:

$$= 6 \times \tfrac{1}{2} \times \frac{\left(2-\sqrt{3}\right)^2 a^2}{3} \times \tfrac{\sqrt{3}}{2}$$

$$= \left(2 - \sqrt{3}\right)^2 a^2 \times \tfrac{\sqrt{3}}{2}$$

$$= \left(7 - 4\sqrt{3}\right)a^2 \times \tfrac{\sqrt{3}}{2}$$

$$= \left(\tfrac{7}{2}\sqrt{3} - 6\right)a^2$$

Q7.

$$1,000 \times \left(1+\tfrac{x}{100}\right)^{50} = 10,000 \times \left(1+\tfrac{y}{100}\right)^{50}$$

$$\left(1+\tfrac{x}{100}\right)^{50} = 10\left(1+\tfrac{y}{100}\right)^{50}$$

$$1+\tfrac{x}{100} = \sqrt[50]{10}\left(1+\tfrac{y}{100}\right)$$

$$\tfrac{x}{100} = \sqrt[50]{10}\left(1+\tfrac{y}{100}\right) - 1$$

$$x = \sqrt[50]{10}(100+y) - 100$$

Q8.

Cost of coffee $= C$

Cost of sandwich $= S$

$3C + 2S = 14.40$* (multiply by 11)

After price increase the cost of a coffee will be $1.1C$ and cost of sandwich $= 1.1S$

$2.2C + 1.1S = 9.13$ (multiply by 20)

$33C + 22S = 158.40$

$44C + 22S = 182.60$

$11C = 24.20$

$C = 2.20$

Sub into * gives $S = 3.90$

Q9.

$$6^x = \left(6^{50} + 9\right)^2 - \left(6^{50} - 9\right)^2$$

$$6^x = \left(6^{50} + 9 + 6^{50} - 9\right) \times \left(6^{50} + 9 - \left(6^{50} - 9\right)\right)$$

$$6^x = \left(2 \times 6^{50}\right) \times 18$$

$$6^x = 36 \times 6^{50}$$

$$6^x = 6^2 \times 6^{50}$$

$$6^x = 6^{52}$$

$$x = 52$$

Q10.

The listing of outcomes here is extensive but is shortened significantly if you group the possible last rolls.

The possible rolls were:

$(6, 5, 4 - 1), (6, 4, 3 - 1), (6, 3, 2 - 1), (6, 2, 1)$

Thus the probability P is given by:

$$\begin{aligned} P &= \left(\tfrac{1}{6} \times \tfrac{4}{6}\right) + \left(\tfrac{1}{6} \times \tfrac{3}{6}\right) + \left(\tfrac{1}{6} \times \tfrac{2}{6}\right) + \left(\tfrac{1}{6} \times \tfrac{1}{6}\right) \\ &= \tfrac{4}{36} + \tfrac{3}{36} + \tfrac{2}{36} + \tfrac{1}{36} \\ &= \tfrac{10}{36} \\ &= \tfrac{5}{18} \end{aligned}$$

Q11.

Total chocolates $= m + n$ which must be an even number.

For there to be more in the second box it must contain at least $\frac{m+n+2}{2}$ chocolates.

So number of chocolates needed to moved is:

$= \frac{m+n+2}{2} - \frac{2m}{2}$

$= \frac{n+2-m}{2}$

Q12.

There are 9 such pairs, demonstrated by video solution.

Pairs are: (9, 11), (18, 22), (27, 33) etc.

Q13.

Diagonal of square $= \sqrt{2^2 + 2^2}$

$$= 2\sqrt{2}$$

Applying cosine rule:

$$a^2 = \left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 - \left(2 \times \sqrt{2} \times \sqrt{2} \times \cos(45)\right)$$

$$= 2 + 2 - 4\cos(45)$$

$$= 4 - 4\left(\frac{\sqrt{2}}{2}\right)$$

$$= 4\left(1 - \frac{\sqrt{2}}{2}\right)$$

$$a = 2\sqrt{\frac{2-\sqrt{2}}{2}}$$

Q14.

Sum of radii $= r + ar = 8$

$$r(1+a) = 8$$

$$r = \frac{8}{1+a}$$

Sum of surface areas $= 4\pi r^2 + 4\pi(ar)^2 = 160\pi$

$$160\pi = 4\pi r^2 + 4\pi a^2 r^2$$

$$40 = r^2 + a^2 r^2$$

$$40 = r^2\left(1+a^2\right)$$

$$r^2 = \frac{40}{1+a^2}$$

Equating the two expressions:

$$\left(\frac{8}{1+a}\right)^2 = \frac{40}{1+a^2}$$

$$\frac{64}{1+2a+a^2} = \frac{40}{1+a^2}$$

$$64\left(1+a^2\right) = 40\left(1+2a+a^2\right)$$

$$0 = 40\left(1+2a+a^2\right) - 64\left(1+a^2\right)$$

$$0 = 40 + 80a + 40a^2 - 64 - 64a^2$$

$$0 = -24a^2 + 80a - 24$$

$$0 = 3a^2 - 10a + 3$$

$$0 = (3a-1)(a-3)$$

Therefore the radius of the larger sphere is 3 times the radius of the smaller sphere.

Subbing back into earlier equations to find the radii:

$$r + 3r = 8$$

$$4r = 8$$

$r = 2$ (smaller) or $r = 6$ (larger).

Volume of larger sphere, V:

$$V = \frac{4}{3}\pi(6)^3$$

$$= 288\pi$$

Q15.

To calculate the number of half lives have passed since the first reading:

$1 \div 0.00390625 = 256$

So count rate is 256 times smaller at the second reading than at the first reading. Thus 8 half lives had passed since $2^8 = 256$.

$8 \times 22 = 176$ minutes

$176 = 60 + 60 + 56$

So time of second reading was 2.56pm.

Q16.

Call the largest angle, A, then:

$$P(A > 90) = \frac{3}{20}$$

Click here for explanation of result.

Q17.

$$Area \; = \; 8\pi \, cm^2$$

See video.

Q18.

$$a^4 = \frac{2581 - b^4}{4}$$

$$4a^4 + b^4 = 2581$$

$$4a^4 + b^4 = 29 \times 89$$

$$4a^4 + b^4 = \left(a^2 + (a+b)^2\right)\left(a^2 + (a-b)^2\right)$$

So...

$$2581 = \left(a^2 + (a+b)^2\right)\left(a^2 + (a-b)^2\right)$$

$$29 \times 89 = \left(a^2 + (a+b)^2\right)\left(a^2 + (a-b)^2\right)$$

$$(25 + 64)(25 + 4) = \left(a^2 + (a+b)^2\right)\left(a^2 + (a-b)^2\right)$$

$$\left(5^2 + (5+3)^2\right)\left(5^2 + (5-3)^2\right) = \left(a^2 + (a+b)^2\right)\left(a^2 + (a-b)^2\right)$$

So...

$$a = 5, b = 3$$